$(A - \lambda E)\, h_2 = h_1$

$(A - E)\, h_2 = h_1$

$\begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} h_{21} \\ h_{22} \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\quad 2h_{21} - h_{22} = 1$

$h_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$T = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$

$J = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$e^{Jt} = \begin{pmatrix} e^t & te^t \\ 0 & e^t \end{pmatrix}$

$X = T \cdot e^{Jt} \cdot c =$

$= \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} e^t & te^t \\ 0 & e^t \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} =$

$= \Bigg(\underbrace{\boxed{\begin{matrix} e^t \\ 2e^t \end{matrix}}}_{e_1} \; \underbrace{\boxed{\begin{matrix} -te^t + e^t \\ 2te^t + e^t \end{matrix}}}_{e_2} \Bigg) \cdot c$ (iz I načina)

Za vježbu: $x_1' = -3x_1 + 2x_2$

$x_2' = -2x_1 + x_2$

5. $x_1' = x_1 + x_2$

$x_2' = -2x_1 + 3x_2$

R

$A = \begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix}$

$\det(A - \lambda E) = \begin{vmatrix} 1-\lambda & 1 \\ -2 & 3-\lambda \end{vmatrix} = 3 - 4\lambda + \lambda^2 + 2$

$\lambda^2 - 4\lambda + 5 = 0 \Rightarrow \lambda_{1/2} = \dfrac{4 \pm \sqrt{16-20}}{2} = 2 \pm i$

$\lambda_1 = 2+i$

$(A-(2+i)E)\,h_1 = \vec{0}$

$$\begin{pmatrix} -1-i & 1 \\ -2 & 1-i \end{pmatrix}\begin{pmatrix} h_{11} \\ h_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow (-1-i)h_{11} + h_{12} = 0$$

$$h_{12} = (1+i)\,h_{11}$$

$$h_1 = \begin{pmatrix} 1 \\ 1+i \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{h_1^1} + i\underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{h_1^2}$$

// Komentar

$\alpha + i\beta \longrightarrow h_1^1 + i h_1^2$

$$(h_1^1 + i h_1^2)e^{(\alpha+i\beta)t} = (h_1^1 + i h_1^2)e^{\alpha t}(\cos\beta t + i\sin\beta t) =$$

$$= \boxed{e^{\alpha t}(h_1^1\cos\beta t - h_1^2\sin\beta t)} +$$

$$+ i\boxed{e^{\alpha t}(h_1^2\cos\beta t + h_1^1\sin\beta t)}$$

ova dva reš. su nezavisna

$\overline{A h_1} = \overline{\lambda_1 h_1}$

$A\overline{h_1} = \overline{\lambda_1}\,\overline{h_1}$
$\quad\;\downarrow$
$\quad\;\lambda_2$

$$\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} + i\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)e^{(2+i)t} = \boxed{e^{2t}\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\cos t - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\sin t\right)} - \varphi_1 \; +$$

$$+ i\boxed{e^{2t}\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\cos t + \begin{pmatrix} 1 \\ 1 \end{pmatrix}\sin t\right)} - \varphi_2$$

φ_1, φ_2 - nez. reš.

$X = c_1\varphi_1 + c_2\varphi_2$ opšte rešenje

$e^{x+iy} = e^x(\cos y + i\sin y)$

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⑥ $x_1' = 2x_1 + 2x_3 - x_2$

$x_2' = x_1 + 2x_3$

$x_3' = x_2 - 2x_1 - x_3$

$$A = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 0 & 2 \\ -2 & 1 & -1 \end{pmatrix}$$

$$\det(A - \lambda E) = \begin{vmatrix} 2-\lambda & -1 & 2 \\ 1 & -\lambda & 2 \\ -2 & -1 & -1-\lambda \end{vmatrix} =$$

$$= (2-\lambda)(\lambda(1+\lambda) - 2) + 1(-1-\lambda+4) + 2(1-2\lambda) =$$

$$= -(\lambda^3 - \lambda^2 + \lambda - 1) = -(\lambda^2(\lambda-1) + \lambda - 1) = -(\lambda-1)(\lambda^2+1)$$

$\lambda_1 = 1 \qquad \lambda_{2,3} = \pm i$

$(A - E)h_1 = 0$

$$\begin{pmatrix} 1 & -1 & 2 \\ 1 & -1 & 2 \\ -2 & 1 & -2 \end{pmatrix}\begin{pmatrix} h_{11} \\ h_{12} \\ h_{13} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad e_1 = h_1 e^t \qquad \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = h_1$$

$\lambda_2 = i$

$$\begin{pmatrix} 2-i & -1 & 2 \\ 1 & -i & 2 \\ -2 & 1 & -1-i \end{pmatrix}\begin{pmatrix} h_{21} \\ h_{22} \\ h_{23} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$h_2 = \begin{pmatrix} 1 \\ 1 \\ \frac{i-1}{2} \end{pmatrix}$$

$$\underset{(\cos t + i\sin t)}{e^{it}}\left(\begin{pmatrix} 1 \\ 1 \\ -\frac{1}{2} \end{pmatrix} + i\begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}\right) = \begin{pmatrix} \cos t + i\sin t \\ \cos t + i\sin t \\ \frac{1}{2}(i\cos t - \cos t - \sin t - i\sin t) \end{pmatrix}$$

$$\mathrm{Re}\cdot \begin{pmatrix} \cos t \\ \cos t \\ -\frac{1}{2}\cos t - \frac{1}{2}\sin t \end{pmatrix} \longrightarrow e_2$$

$$\text{lu:}\ \begin{pmatrix} \sin t \\ \sin t \\ \frac{\cos t}{2} - \frac{\sin t}{2} \end{pmatrix} \longrightarrow e_3$$

$$X = c_1 e_1 + c_2 e_2 + c_3 e_3$$

7. $x_1' = 2x_1 - x_2 - x_3$

$x_2' = 3x_1 - 2x_2 - 3x_3$

$x_3' = -x_1 + x_2 + 2x_3$

R:

$\lambda_1 = 0 \longrightarrow (1, 3, -1)$

$\lambda_{2,3} = 1$

$$A - E = \begin{pmatrix} 1 & -1 & -1 \\ 3 & -3 & -3 \\ -1 & 1 & 1 \end{pmatrix} \quad \text{rang} = 1$$

$3 - 1 = 2$ nez. vektora

$$\begin{pmatrix} 1 & -1 & -1 \\ & \dots & \end{pmatrix} \begin{pmatrix} h_{21} \\ h_{22} \\ h_{23} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$h_{21} = h_{22} + h_{23}$$

$$h_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},\ h_3 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \text{- dva nez. vek.}$$

$$T = \begin{pmatrix} 1 & 1 & 0 \\ 3 & 0 & 1 \\ -1 & 1 & -1 \end{pmatrix}$$

$$J = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$e^{Jt} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^t & 0 \\ 0 & 0 & e^t \end{pmatrix} \qquad X = T \cdot Y$$

$$X = T e^{Jt} \cdot c = \begin{pmatrix} 1 & e^t & 0 \\ 3 & 0 & e^t \\ -1 & e^t & -e^t \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

8. $x_1' = 4x_1 - x_2$

$x_2' = 3x_1 + x_2 - x_3$

$x_3' = x_1 + x_3$

R

$\lambda_1 = \lambda_2 = \lambda_3 = 2$

$\longrightarrow \ -(\lambda - 2)^3 = 0$

$$\operatorname{rang}(A - 2E) = \operatorname{rang} \left(\begin{array}{cc|c} 2 & -1 & 0 \\ 3 & -1 & -1 \\ \hline 1 & 0 & -1 \end{array} \right) = 2$$

$3 - 2 = 1$ nez. w. vek.

$2h_{11} - h_{12} = 0 \Rightarrow h_{12} = 2h_{11} \quad (1)$

$3h_{11} - h_{12} - h_{13} = 0 \quad (2)$

$(2) \Rightarrow h_{13} = 3h_{11} - 2h_{11} = h_{11}$

$$h_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$h_2 = ?$

$(A - 2E)h_2 = h_1$

$$\begin{cases} 2h_{21} - h_{22} = 1 \\ 3h_{21} - h_{22} - h_{23} = 2 \end{cases}$$

$h_{22} = 2h_{21} - 1$

$h_{23} = 3h_{21} - h_{22} - 2 = 3h_{21} - 2h_{21} + 1 - 2 = h_{21} - 1$

$$h_{21} = 1 \qquad h_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$(A-2E)h_3 = h_2$

$h_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\quad T = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

$J = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ $\quad e^{Jt} = \begin{pmatrix} e^{2t} & te^{2t} & \frac{t^2}{2!}e^{2t} \\ 0 & e^{2t} & te^{2t} \\ 0 & 0 & e^{2t} \end{pmatrix}$

$x = T \cdot e^{Jt} \cdot c$

Zad.

$x_1' = x_1 - x_2 + x_3$

$x_2' = x_1 + x_2 - x_3$

$x_3' = 2x_3 - x_2$

$\lambda_1 = 2 \longrightarrow \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = h_1$

$\lambda_{2,3} = 1 \longrightarrow \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = h_2$ svojstveni

$(A-E)h_3 = h_2$

$h_3 = \begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix}$

$T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -2 \\ 1 & 1 & -1 \end{pmatrix}, \quad J = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

$e^{Jt} = \begin{pmatrix} e^{2t} & 0 & 0 \\ 0 & e^{t} & te^{t} \\ 0 & 0 & e^{t} \end{pmatrix}$

$x = T \cdot e^{Jt} \cdot c$

Nehomogeni linearni sistemi d-j

Metod varijacije konstanti:

$X' = \overset{(n)}{A}X + B(t)$

$X_H = \phi(t) \cdot C$ (znamo riješiti)

$X' = \phi' \cdot c + \phi \cdot c'$

$\cancel{\phi' c} + \phi \cdot c' = \cancel{A\phi \cdot c} + B(t)$ $\quad \Big| \; \big(\underbrace{\phi' - A\phi}_{0}\big)c$

$c' = \phi^{-1}_{(t)} B(t) \;/\int$

$c(t) = \int \phi^{-1}(t) B(t) dt + \gamma$

$X = \phi(t) \cdot \gamma + \phi(t) \int \phi^{-1}(t) B(t) dt$

Desna strana Kvazipolinom (za sisteme sa konstantnim koef.)

$B(t) = e^{\gamma t} R_m(t) = e^{\gamma t}(p_m t^m + \dots + p_0)$, $\gamma \in \mathbb{R}$, $p_i \in \mathbb{R}^n$

s višestrukost za γ

$s = 0$ $\quad x_p = Q_m(t) e^{\gamma t}$

$s \neq 0$ $\quad x_p = Q_{m+s}(t) e^{\gamma t}$

Ako je $B(t) = e^{\alpha t}(P_m(t)\cos(\beta t) + Q_n(t)\sin(\beta t))$

s višestrukost za $\alpha + i\beta$

$x_p(t) = e^{\alpha t}(H_{k+s}(t)\cos(\beta t) + R_{k+s}(t)\sin(\beta t))$

$k = \max\{m, n\}$.

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(1.) $x_1' = x_2 + 2e^t$

$x_2' = x_1$

R

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$$

$$\lambda_1 = 1 \longrightarrow h_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\lambda_2 = -1 \longrightarrow h_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\Phi(t) = \begin{pmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{pmatrix} \qquad \det \phi(t) = -1 - 1 = -2$$

$$B(t) = \begin{pmatrix} 2e^t \\ 0 \end{pmatrix}$$

$$\Phi^{-1}(t) = \frac{1}{-1-1} \begin{pmatrix} -e^{-t} & -e^t \\ -e^{-t} & e^t \end{pmatrix}^T = \begin{pmatrix} \frac{e^{-t}}{2} & \frac{e^{-t}}{2} \\ \frac{e^t}{2} & \frac{e^t}{-2} \end{pmatrix}$$

$$\Phi^{-1}(t) \cdot B(t) = \frac{1}{2} \begin{pmatrix} e^{-t} & e^{-t} \\ e^{+t} & -e^t \end{pmatrix} \begin{pmatrix} 2e^t \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 \\ 2e^{2t} \end{pmatrix} = \begin{pmatrix} 1 \\ e^{2t} \end{pmatrix}$$

$$\int \Phi^{-1} B(t)\, dt = \int \begin{pmatrix} 1 \\ e^{2t} \end{pmatrix} dt = \begin{pmatrix} t \\ \frac{1}{2} e^{2t} \end{pmatrix}$$

$$X = \Phi(t) k + \Phi(t) \begin{pmatrix} t \\ \frac{1}{2} e^{2t} \end{pmatrix} =$$

$$= \begin{pmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} + \begin{pmatrix} te^t + \frac{1}{2} e^t \\ te^t - \frac{1}{2} e^t \end{pmatrix}$$

(2) $x_1' = 5x_1 - 3x_2 + 2e^{3t}$

$x_2' = x_1 + x_2 + 5e^{-t}$

B(t)

R

$\lambda = 2 \longrightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda = 4 \longrightarrow \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$\phi(t) = \begin{pmatrix} e^{2t} & 3e^{4t} \\ e^{2t} & e^{4t} \end{pmatrix}$

$\phi^{-1}(t) = \begin{pmatrix} -\frac{1}{2}e^{-2t} & \frac{3}{2}e^{-2t} \\ \frac{1}{2}e^{-4t} & -\frac{1}{2}e^{-4t} \end{pmatrix}$

$\int \underbrace{\phi^{-1}(t) B(t)}_{F} = \begin{pmatrix} -e^{t} - \frac{5}{2}e^{-3t} \\ -e^{t} + \frac{1}{2}e^{-5t} \end{pmatrix}$

$\phi \cdot F = \begin{pmatrix} -4e^{3t} - e^{-t} \\ -2e^{3t} - 2e^{-t} \end{pmatrix} = P$

$X = \phi(t) \cdot C + P$

① $\begin{cases} x_1' = 2x_2 - x_1 + 1 \\ x_2' = 3x_2 - 2x_1 \end{cases}$ $\Leftrightarrow$ $\underline{X' = AX + B(t)}$

$A = \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}$ $B(t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} t^0 \cdot e^{0t}$

$\lambda_{1,2} = 1$

$\mu = 0,\ m = 0 \Rightarrow s = 0$

$X_p(t) = C$

$0 = AC + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\Rightarrow$ $\begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = A^{-1}\begin{pmatrix} -1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

Kij je $X = \phi(t) \cdot \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

② $\begin{cases} x_1' = 2x_1 - x_2 \\ x_2' = x_2 - 2x_1 + 18t \end{cases}$ $X' = AX + B(t)$.

$\lambda_1 = 0 \mapsto \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\lambda_2 = 3 \mapsto \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$X_H = c_1 e^{0t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \cdot e^{3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & e^{3t} \\ 2 & -e^{3t} \end{pmatrix}}_{\phi(t)}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

$B(t) = \begin{pmatrix} 0 \\ 18 \end{pmatrix} t e^{0 \cdot t}$

$X_p = a_2 t^2 + a_1 t + a_0$

$X_p' = 2a_2 t + a_1$

$m = 1$

$s = 1$ jer $\mu = 0$.

$2a_2 t + a_1 = A(a_2 t^2 + a_1 t + a_0) + \begin{pmatrix} 0 \\ 18 \end{pmatrix} t$

$\begin{cases} Aa_2 = 0 \\ Aa_1 + \begin{pmatrix} 0 \\ 18 \end{pmatrix} = 2a_2 \\ Aa_0 = a_1 \end{cases}$

$A = \begin{pmatrix} 2 & -1 \\ -2 & 1 \end{pmatrix}$

$\boxed{2a_{21} - a_{22} = 0}$ (1)

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$$\begin{pmatrix} 2 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} a_{11} \\ a_{12} \end{pmatrix} = \begin{pmatrix} 2a_{21} \\ 2a_{22}-18 \end{pmatrix} \qquad \begin{cases} 2a_{11} - a_{12} = 2a_{21} \\ -2a_{11} + a_{12} = 2a_{22} - 18 \end{cases}$$

Da bi sistem imao rješenja, mora biti

$$\boxed{2a_{21} = -(2a_{22} - 18)} \quad (2)$$

Iz (1) i (2) $\quad a_{22} = 2a_{21}$

$$2a_{21} + 4a_{21} = 18 \Rightarrow \begin{matrix} a_{21} = 3 \\ a_{22} = 6 \end{matrix} \Rightarrow a_2 = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

$$\boxed{2a_{11} - a_{12} = 6} \quad (3)$$

$$\begin{pmatrix} 2 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} a_{01} \\ a_{02} \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{12} \end{pmatrix} \Rightarrow \boxed{a_{11} = -a_{12}} \quad (4)$$

Iz (3) i (4) $\quad \begin{matrix} a_{11} = 2 \\ a_{12} = -2 \end{matrix} \quad \Rightarrow a_1 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}.$

Ostaje još $\quad 2a_{01} - a_{02} = 2 \quad$ i npr $\begin{matrix} a_{01} = 1 \\ a_{02} = 0. \end{matrix}$ (ima još rješenja)

$$x_p = \begin{pmatrix} 3 \\ 6 \end{pmatrix} t^2 + \begin{pmatrix} 2 \\ -2 \end{pmatrix} t + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Rj. je $\quad x = x_H + x_p$

3 $\quad \begin{cases} x_1' = 2x_1 - 3x_2 \\ x_2' = x_1 - 2x_2 + 2\sin t \end{cases}$

Rj

$$A = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix} \qquad \begin{matrix} \lambda_1 = 1 \longmapsto \begin{pmatrix} 3 \\ 1 \end{pmatrix} = h_1 \\ \lambda_2 = -1 \longmapsto \begin{pmatrix} 1 \\ 1 \end{pmatrix} = h_2 \end{matrix}$$

$$x_H = c_1 e^t \begin{pmatrix} 3 \\ 1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3e^t & e^{-t} \\ e^t & e^{-t} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$B(t) = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \sin t$

$\alpha = 0, \beta = 1 \Rightarrow s = 0$

$n = m = 0.$

$x_p(t) = a\cos t + b\sin t$

$x_p'(t) = -a\sin t + b\cos t.$

$X' = AX + B(t).$

$-a\sin t + b\cos t = (Aa)\cos t + (Ab)\sin t + \begin{pmatrix} 0 \\ 2 \end{pmatrix} \sin t.$

$\begin{cases} Aa = b. & /A \quad Ab = \underset{E}{\overset{\shortparallel}{A^2}} a = a \qquad Ab = a \\ Ab + \begin{pmatrix} 0 \\ 2 \end{pmatrix} = -a \end{cases}$

$Ab + \begin{pmatrix} 0 \\ 2 \end{pmatrix} = -Ab \Rightarrow 2Ab = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \Rightarrow Ab = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = a$

$b = A^{-1}\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$x_p = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \sin t + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \cos t.$

$X = X_H + X_p$

④ $\begin{cases} x_1' = 2x_1 - x_2 \\ x_2' = 2x_2 - x_1 - 5e^t \sin t \end{cases}$

④ $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$

$\lambda_1 = 1 \longmapsto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\qquad B(t) = \begin{pmatrix} 0 \\ -5 \end{pmatrix} e^t \sin t$

$\lambda_2 = 3 \longmapsto \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\alpha = 1, \beta = 1 \Rightarrow s = 0.$

$n = m = 0$

$x_p(t) = e^t(a\cos t + b\sin t)$

$x_p' = e^t(a\cos t + b\sin t) + e^t(-a\sin t + b\cos t)$

$X' = AX + B(t)$

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$e^t(a\cos t + b\sin t - a\sin t + b\cos t) =$

$= A(a\cos t + b\sin t)\, \cancel{e^t} + \binom{0}{5}\cancel{e^t} \sin t$

$/A-E$

$$\begin{cases} Aa = a + b \\ Ab = b - a + \binom{0}{5} \end{cases} \Leftrightarrow \begin{aligned} (A-E)a &= b \\ (A-E)b &= \binom{0}{5} - a \end{aligned}$$

$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ $\qquad \boxed{b_1 = -b_2}$

$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} -a_1 \\ 5 - a_2 \end{pmatrix}$ $\qquad \boxed{a_1 = 5 - a_2}$

$\boxed{a_1 - a_2 = b_1}$ $\qquad a_1 - (5 - a_1) = b_1 \Leftrightarrow 2a_1 - b_1 = 5$

$\boxed{b_1 - b_2 = -a_1}$ $\qquad b_1 + b_1 = -a_1 \Leftrightarrow a_1 + 2b_1 = 0.$

$\Downarrow$

$5a_1 = 10 \quad a_1 = 2$

$b_1 = -1$

$\cancel{a_2 = 3}$

$\boxed{\begin{aligned} a_2 &= 3 \\ b_2 &= 1 \end{aligned}}$

Ђ. $a = \binom{2}{3} \quad b = \binom{-1}{1}$

$x_p = e^t\left(\binom{-1}{1}\sin t + \binom{2}{3}\cos t\right)$

$X = x_H + x_p$, opšte R.

(Zad) $x_1' = 2x_1 - x_2 - x_3 + e^{2t}$
$x_2' = 3x_1 - 2x_2 - 3x_3 + \cos t$
$x_3' = -x_1 + x_2 + 2x_3 + e^t$

ili $x' = Ax + \begin{pmatrix}1\\0\\0\end{pmatrix}e^{2t} + \begin{pmatrix}0\\1\\0\end{pmatrix}\cos t + \begin{pmatrix}0\\0\\1\end{pmatrix}e^t$

(R)

$\lambda_1 = 0$, $\lambda_{2,3} = 1$ } Rijesiti homogeni dio.

$x_{p1} \to b_1(t) = \begin{pmatrix}1\\0\\0\end{pmatrix}e^{2t}$, $s=0$ $(\gamma = 2)$, $m=0$

$x_{p1} = ae^{2t}$, $a = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$

$2ae^{2t} = (Aa)e^{2t} + \begin{pmatrix}1\\0\\0\end{pmatrix}e^{2t}$

$(A-2E)a = \begin{pmatrix}-1\\0\\0\end{pmatrix}$ $\quad a = (A-2E)^{-1}\begin{pmatrix}-1\\0\\0\end{pmatrix} = \dots$

$x_{p2} \to b_2(t) = \begin{pmatrix}0\\1\\0\end{pmatrix}\cos t$. $\overset{\alpha=0,\beta=1}{(\gamma = i)} \Rightarrow s=0$.

$x_{p2}(t) = b\cos t + c\sin t$.

$-b\sin t + c\cos t = A(b\cos t + c\sin t) + \begin{pmatrix}0\\1\\0\end{pmatrix}\cos t$

$\begin{cases} Ab + \begin{pmatrix}0\\1\\0\end{pmatrix} = c \quad /\cdot A \\ Ac = -b \end{cases}$

$A^2b + A\begin{pmatrix}0\\1\\0\end{pmatrix} = Ac = -b$

$(A^2+E)b = -A\begin{pmatrix}0\\1\\0\end{pmatrix}$

$b = (A^2+E)^{-1}\left(-A\begin{pmatrix}0\\1\\0\end{pmatrix}\right)$

$c = Ab + \begin{pmatrix}0\\1\\0\end{pmatrix}$

x_{p3} $\quad \gamma = 1 \Rightarrow s = 2$

$x_{p3}(t) = (dt^2 + et + f)e^t$

$e^t(dt^2 + et + f + 2dt + e) = ((Ad)t^2 + (Ae)t + Af)e^t + \begin{pmatrix}0\\0\\1\end{pmatrix}e^t$

$\begin{cases} Ad = d \\ Ae = 2d + e \\ Af + \begin{pmatrix}0\\0\\1\end{pmatrix} = f + e. \end{cases}$

$x = x_H + x_{p1} + x_{p2} + x_{p3}.$